SUBJECT :- MATHEMATICS

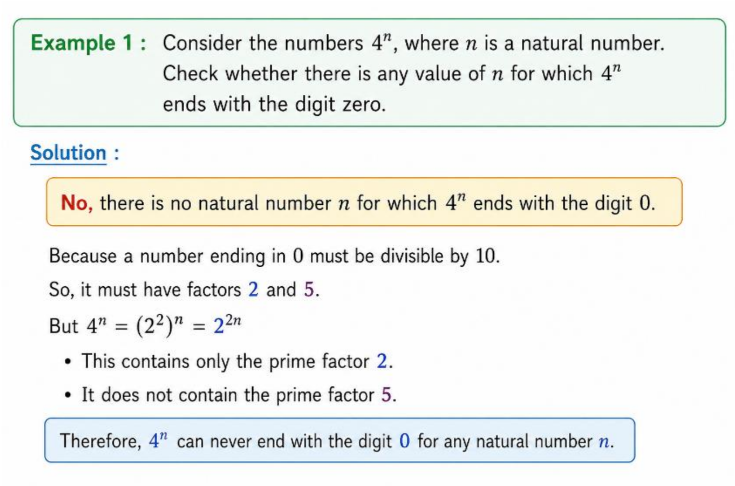
STD:-10TH

CHAPTER:-1 REAL NUMBERS

Example 1 : Consider the numbers 4n , where n is a natural number. Check whether there is any value of n for which 4n ends with the digit zero.

ANS:- No, there is no natural number n for which 4^n ends with the digit 0.

Because a number ending in 0 must be divisible by 10, so it must have factors 2 and 5. But $4n=(2^2)^n=2^{2n}$ contains only factor 2 and no factor 5. Therefore, 4^n can never end with 0.



Example 1 : Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Solution :

No, there is no natural number n for which 4^n ends with the digit 0.

Because a number ending in 0 must be divisible by 10.

So, it must have factors 2 and 5.

But $4^n = (2^2)^n = 2^{2n}$

- This contains only the prime factor 2.
- It does not contain the prime factor 5.

Therefore, 4^n can never end with the digit 0 for any natural number n.

Example 2 : Find the LCM and HCF of 6 and 20 by the prime factorisation method.

ANS:- Prime factorisation:

6 = 2 × 3
20 = 2 × 2 × 5

HCF = 2
LCM = 2 × 2 × 3 × 5 = 60

Example 3 : Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

ANS:- $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$
$404 = 2 \times 2 \times 101 = 2^2 \times 101$

HCF = 2^2 = 4

LCM = (96 × 404) ÷ 4 = 9696

∴ HCF = 4 and LCM = 9696.

Example 4 : Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

ANS:- Prime factorisation:

$6 = 2 \times 3$
$72 = 2^3 \times 3^2$
$120 = 2^3 \times 3 \times 5$

HCF = Product of common primes with smallest powers
= 2 × 3 = 6

LCM = Product of all primes with greatest powers
$= 2^3 \times 3^2 \times 5$
= 8 × 9 × 5
= 360

∴ HCF = 6 and LCM = 360.

EXERCISE 1.1

1. Express each number as a product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

ANS:- (i) $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) $5005 = 5 \times 7 \times 11 \times 13$

(v) $7429 = 17 \times 19 \times 23$

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

(i) 26 and 91

$26 = 2 \times 13$, $91 = 7 \times 13$
HCF = 13
LCM = $2 \times 7 \times 13 = 182$

Verification:
LCM × HCF = $182 \times 13 = 2366$
Product = $26 \times 91 = 2366$ ✔

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$, $92 = 2^2 \times 23$
HCF = 2
LCM = $2^2 \times 3 \times 5 \times 17 \times 23 = 23460$

Verification:
LCM × HCF = $23460 \times 2 = 46920$
Product = $510 \times 92 = 46920$ ✔

(iii) 336 and 54

$336 = 2^4 \times 3 \times 7$, $54 = 2 \times 3^3$
HCF = $2 \times 3 = 6$
LCM = $2^4 \times 3^3 \times 7 = 3024$

Verification:
LCM × HCF = $3024 \times 6 = 18144$
Product = $336 \times 54 = 18144$ ✔

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

ANS:- (i) 12, 15 and 21

$12 = 2^2 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$

HCF = 3
LCM = $2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29

17, 23, 29 are all prime numbers

HCF = 1
LCM = 17 × 23 × 29 = 11339

(iii) 8, 9 and 25

$8 = 2^3$
$9 = 3^2$
$25 = 5^2$

HCF = 1
LCM = $2^3 \times 3^2 \times 5^2 = 1800$

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

ANS:- Given:
HCF (306, 657) = 9

We use the formula:
LCM × HCF = Product of the numbers

So,
LCM = (306 × 657) ÷ 9

= 306 × 73
= 22338

Final Answer:

LCM (306, 657) = 22338 ✔

5. Check whether 6^n can end with the digit 0 for any natural number n.

ANS:- To end with digit **0**, a number must be divisible by **10 = 2 × 5**, so it must contain factor **5**.

Now,
$6^n = (2\times3)^n = 2^n\times3^n$

There is **no factor 5** in 6^n, so it can never be divisible by 10.

Final Answer:

No, 6^n cannot end with the digit 0 for any natural number n. ✔

6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

ANS:-(i) 7×11×13+13

Take 13 common:
$=13(7\times11+1)$

$=13(77+1)=13\times78$

Since it is a product of two numbers greater than 1, it is **composite**.

(ii) $7\times6\times5\times4\times3\times2\times1+5$

$=7!+5$

Take 5 common:
$=5(7!/5+1)$

Since it has factors other than 1 and itself, it is **composite**.

Final Answer:

Both numbers can be factorised into products of integers greater than 1, so both are **composite numbers.**

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

ANS:- To find when they meet again at the starting point, we take the **LCM of their time periods**.

Sonia = 18 minutes
Ravi = 12 minutes

Prime factors:
$18 = 2 \times 3^2$
$12 = 2^2 \times 3$

LCM = $2^2 \times 3^2$ = 36 minutes

Final Answer:

They will meet again at the starting point after **36 minutes** ✔

Example 5 : Prove that root √3 is irrational.

ANS:- Proof that √3 is irrational:

Assume that $\sqrt{3}$ is rational. Then we can write:
$\sqrt{3} = a/b$, where a and b are co-prime integers and $b \neq 0$.

Squaring both sides:
$3 = a^2/b^2$
$\Rightarrow a^2 = 3b^2$

So, a^2 is divisible by 3 $\Rightarrow$ a is also divisible by 3.
Let $a = 3k$.

Substitute:
$(3k)^2 = 3b^2$
$\Rightarrow 9k^2 = 3b^2$
$\Rightarrow b^2 = 3k^2$

So, b^2 is divisible by 3 $\Rightarrow$ b is also divisible by 3.

Thus, both a and b have a common factor 3, which contradicts that they are co-prime.

Conclusion:

Our assumption is false. Therefore, **$\sqrt{3}$ is irrational** ✔

Example 6: Show that $5 - \sqrt{3}$ is irrational.

ANS:- Proof that $5-\sqrt{3}$ is irrational:

Assume $5-\sqrt{3}$ is rational.

Let
$5-\sqrt{3}=r$, where r is rational.

Then,
$3=5-r$

Now, 5 is rational and r is rational, so $5-r$ is also rational.

This implies $\sqrt{3}$ is rational, which is false (since $\sqrt{3}$ is irrational).

Conclusion:

Our assumption is wrong. Therefore, **$5-\sqrt{3}$ is irrational**.

Example 7 : Show that is irrational. $3\sqrt{2}$ is irrational.

ANS:- Proof:

Assume $3\sqrt{2}$ is rational.

Let
$3\sqrt{2}=r$, where r is rational.

Then,
$\sqrt{2}=\frac{r}{3}$

Since r is rational and 3 is rational, $\frac{r}{3}$ is rational.

So this implies $\sqrt{2}$ is rational, which is false because $\sqrt{2}$ is irrational.

Conclusion:

Our assumption is wrong. Therefore, **$3\sqrt{2}$ is irrational.** ✔

EXERCISE 1.2

1. **Prove that $\sqrt{5}$ is irrational.**

ANS:- Proof that $\sqrt{5}$ is irrational:

Assume that $\sqrt{5}$ is rational.
So we can write:

$\sqrt{5} = a/b$, where a and b are co-prime integers and $b \neq 0$.

Squaring both sides:
$5 = a^2/b^2$
$\Rightarrow a^2 = 5b^2$

So, a^2 is divisible by 5 $\Rightarrow$ a is divisible by 5.
Let $a = 5k$.

Substitute:
$(5k)^2 = 5b^2$
$\Rightarrow 25k^2 = 5b^2$
$\Rightarrow b^2 = 5k^2$

So, b^2 is divisible by 5 $\Rightarrow$ b is divisible by 5.

Thus, both a and b have a common factor 5, which contradicts that they are co-prime.

Conclusion:

Our assumption is false. Therefore, **√5 is irrational** ✔

2. **Prove that 3+ 2√5 is irrational.**

ANS:- Proof that 3+2√5 is irrational:

Assume 3+2√5 is rational.

Let
3+2√5=r ,where r is rational.

Then,
2√5= r-3

So,
$\sqrt{5}=\frac{r-3}{2}$\

Since r and 3 are rational ,$\frac{r-3}{2}$ is rational.

This implies √5 is rational, which is false because √5 is irrational.

Conclusion:

Our assumption is wrong. Therefore, **3+2√5 is irrational.** ✔

3. Prove that the following are irrationals :

(i) $\frac{1}{\sqrt{2}}$ (ii) 7√ 5 (iii) $6+\sqrt{2}$

ANS:- (i) $\frac{1}{\sqrt{2}}$ is irrational.

Assume

$\frac{1}{\sqrt{2}}$ is rational.
Then $\sqrt{2}=\frac{1}{1\sqrt{2}}$would be rational.
But $\sqrt{2}$ is irrational.
So, $\frac{1}{1\sqrt{2}}$ is **irrational**.

(ii) 7√5 is irrational

Assume $7\sqrt{5}$ is rational.

Then $\sqrt{5}=\frac{7\sqrt{5}}{7}$ would be rational.

But $\sqrt{5}$ is irrational.

So, $7\sqrt{5}$ is **irrational**.

THANK YOU